

Number Systems

Exercise 1.1 Page: 5

1. Is zero a rational number? Can you write it in the form p/q where p and q are integers and $q \neq 0$?

Solution:

We know that a number is said to be rational if it can be written in the form p/q , where p and q are integers and $q \neq 0$.

Taking the case of '0',

Zero can be written in the form $0/1, 0/2, 0/3 \dots$ as well as $0/1, 0/2, 0/3 \dots$. Since it satisfies the necessary condition, we can conclude that 0 can be written in the p/q form, where q can either be positive or negative number.

Hence, 0 is a rational number.

2. Find six rational numbers between 3 and 4.

Solution:

There are infinite rational numbers between 3 and 4.

As we have to find 6 rational numbers between 3 and 4, we will multiply both the numbers, 3 and 4, with $6+1 = 7$ (or any number greater than 6)

$$\text{i.e., } 3 \times (7/7) = 21/7$$

and, $4 \times (7/7) = 28/7$. The numbers in between $21/7$ and $28/7$ will be rational and will fall between 3 and 4.

Hence, $22/7, 23/7, 24/7, 25/7, 26/7, 27/7$ are the 6 rational numbers between 3 and 4.

3. Find five rational numbers between $3/5$ and $4/5$.

Solution:

There are infinite rational numbers between $3/5$ and $4/5$.

To find out 5 rational numbers between $3/5$ and $4/5$, we will multiply both the numbers $3/5$ and $4/5$

with $5+1=6$ (or any number greater than 5)

$$\text{i.e., } (3/5) \times (6/6) = 18/30$$

$$\text{and, } (4/5) \times (6/6) = 24/30$$

The numbers in between $18/30$ and $24/30$ will be rational and will fall between $3/5$ and $4/5$.

Hence, $19/30$, $20/30$, $21/30$, $22/30$, $23/30$ are the 5 rational numbers between $3/5$ and $4/5$

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

Solution:

True

Natural numbers- Numbers starting from 1 to infinity (without fractions or decimals)

i.e., Natural numbers = $1, 2, 3, 4, \dots$

Whole numbers – Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers = $0, 1, 2, 3, \dots$

Or, we can say that whole numbers have all the elements of natural numbers and zero.

Every natural number is a whole number; however, every whole number is not a natural number.

(ii) Every integer is a whole number.

Solution:

False

Integers- Integers are set of numbers that contain positive, negative and 0; excluding fractional and decimal numbers.

i.e., integers = $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers = $0, 1, 2, 3, \dots$

Hence, we can say that integers include whole numbers as well as negative numbers.

Every whole number is an integer; however, every integer is not a whole number.

(iii) Every rational number is a whole number.

Solution:

False

Rational numbers- All numbers in the form p/q , where p and q are integers and $q \neq 0$.

i.e., Rational numbers = $0, 19/30, 2, 9/-3, -12/7, \dots$

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers = 0, 1, 2, 3, ...

Hence, we can say that integers include whole numbers as well as negative numbers.

All whole numbers are rational, however, all rational numbers are not whole numbers.

Exercise 1.2 Page: 8

1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

Solution:

True

Irrational Numbers – A number is said to be irrational, if it **cannot** be written in the p/q , where p and q are integers and $q \neq 0$.

i.e., Irrational numbers = π , e , $\sqrt{3}$, $5+\sqrt{2}$, $6.23146\dots$, $0.101001001000\dots$

Real numbers – The collection of both rational and irrational numbers are known as real numbers.

i.e., Real numbers = $\sqrt{2}$, $\sqrt{5}$, $0.102\dots$

Every irrational number is a real number, however, every real number is not an irrational number.

(ii) Every point on the number line is of the form \sqrt{m} where m is a natural number.

Solution:

False

The statement is false since as per the rule, a negative number cannot be expressed as square roots.

E.g., $\sqrt{9} = 3$ is a natural number.

But $\sqrt{2} = 1.414$ is not a natural number.

Similarly, we know that there are negative numbers on the number line, but when we take the root of a negative number it becomes a complex number and not a natural number.

E.g., $\sqrt{-7} = 7i$, where $i = \sqrt{-1}$

The statement that every point on the number line is of the form \sqrt{m} , where m is a natural number is false.

(iii) Every real number is an irrational number.

Solution:

False

The statement is false. Real numbers include both irrational and rational numbers. Therefore, every real number cannot be an irrational number.

Real numbers – The collection of both rational and irrational numbers are known as real numbers.

i.e., Real numbers = $\sqrt{2}$, $\sqrt{5}$, , 0.102...

Irrational Numbers – A number is said to be irrational, if it **cannot** be written in the p/q , where p and q are integers and $q \neq 0$.

i.e., Irrational numbers = π , e , $\sqrt{3}$, $5+\sqrt{2}$, 6.23146.... , 0.101001001000....

Every irrational number is a real number, however, every real number is not irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution:

No, the square roots of all positive integers are not irrational.

For example,

$\sqrt{4} = 2$ is rational.

$\sqrt{9} = 3$ is rational.

Hence, the square roots of positive integers 4 and 9 are not irrational. (2 and 3, respectively).

3. Show how $\sqrt{5}$ can be represented on the number line.

Solution:

Step 1: Let line AB be of 2 unit on a number line.

Step 2: At B, draw a perpendicular line BC of length 1 unit.

Step 3: Join CA

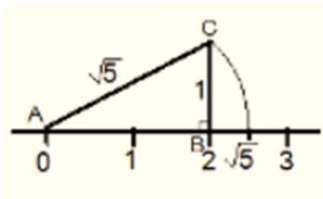
Step 4: Now, ABC is a right angled triangle. Applying Pythagoras theorem,
 $AB^2 + BC^2 = CA^2$

$$2^2 + 1^2 = CA^2 = 5$$

$\Rightarrow CA = \sqrt{5}$. Thus, CA is a line of length $\sqrt{5}$ unit.

Step 4: Taking CA as a radius and A as a center draw an arc touching the number line. The point at which number line get intersected by arc is at $\sqrt{5}$ distance from 0 because it is a radius of the circle whose center was A.

Thus, $\sqrt{5}$ is represented on the number line as shown in the figure.



4. Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP_1 of unit length. Draw a line segment P_1P_2 perpendicular to OP_1 of unit length (see Fig. 1.9). Now draw a line segment P_2P_3 perpendicular to OP_2 . Then draw a line segment P_3P_4 perpendicular to OP_3 . Continuing in Fig. 1.9 :

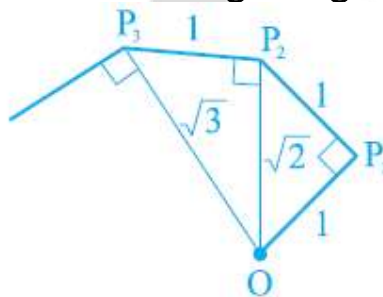
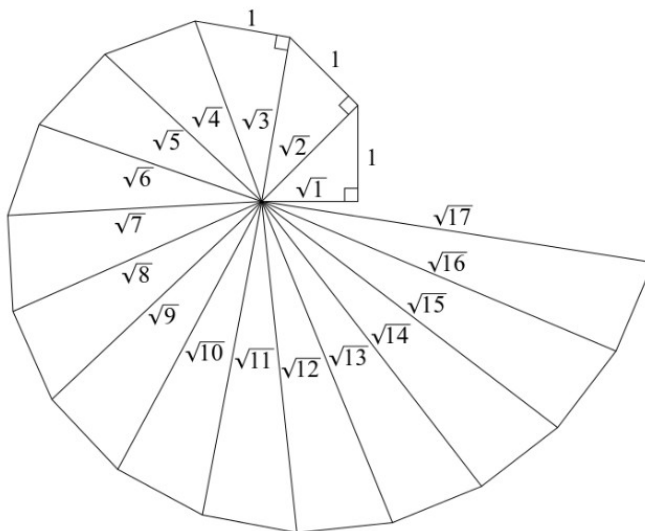


Fig. 1.9 : Constructing square root spiral

Constructing this manner, you can get the line segment $P_{n-1}P_n$ by square root spiral drawing a line segment of unit length perpendicular to OP_{n-1} . In this manner, you will have created the points $P_2, P_3, \dots, P_n, \dots$, and joined them to create a beautiful spiral depicting $\sqrt{2}, \sqrt{3}, \sqrt{4}, \dots$

Solution:



Step 1: Mark a point O on the paper. Here, O will be the center of the square root spiral.

Step 2: From O, draw a straight line, OA, of 1cm horizontally.

Step 3: From A, draw a perpendicular line, AB, of 1 cm.

Step 4: Join OB. Here, OB will be of $\sqrt{2}$

Step 5: Now, from B, draw a perpendicular line of 1 cm and mark the end point C.

Step 6: Join OC. Here, OC will be of $\sqrt{3}$

Step 7: Repeat the steps to draw $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$

Exercise 1.3 Page: 14

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $36/100$

Solution:

$$\begin{array}{r} 00.36 \\ 100 \overline{) 360} \\ \underline{300} \\ 600 \\ \underline{600} \\ 0 \end{array}$$

= 0.36 (Terminating)

(ii) $1/11$

Solution:

$$\begin{array}{r} 0.0909... \\ 11 \overline{) 1} \\ \underline{0} \\ 10 \\ \underline{0} \\ 100 \\ \underline{99} \\ 10 \\ \underline{0} \\ 100 \\ \underline{99} \\ 1 \end{array}$$

= $0.0909... = 0.\overline{09}$ (Non terminating and repeating)

(iii) $4\frac{1}{8}$

Solution:

$$4\frac{1}{8} = \frac{33}{8}$$

$$\begin{array}{r}
 4.125 \\
 8 \overline{) 33} \\
 \underline{32} \\
 10 \\
 \underline{8} \\
 20 \\
 \underline{16} \\
 40 \\
 \underline{40} \\
 0
 \end{array}$$

 $= 4.125$ (Terminating)

(iv) $3/13$

Solution:

$$\begin{array}{r}
 0.230769 \\
 13 \overline{) 30} \\
 \underline{26} \\
 40 \\
 \underline{39} \\
 10 \\
 \underline{0} \\
 100 \\
 \underline{91} \\
 90 \\
 \underline{78} \\
 120 \\
 \underline{117} \\
 3
 \end{array}$$

$= 0.230769... = 0.\overline{230769}$

(v) $2/11$

Solution:

$$\begin{array}{r}
 0.18 \\
 11 \overline{) 2} \\
 \underline{0} \\
 20 \\
 \underline{11} \\
 90 \\
 \underline{88} \\
 2
 \end{array}$$

$$= 0.181818181818... = 0.\overline{18} \text{ (Non terminating and repeating)}$$

(vi) $329/400$

Solution:

$$\begin{array}{r} 400 \overline{) 329} \\ \underline{0} \\ 3290 \\ \underline{3200} \\ 900 \\ \underline{800} \\ 1000 \\ \underline{800} \\ 2000 \\ \underline{2000} \\ 0 \end{array}$$

$= 0.8225$ (Terminating)

2. You know that $1/7 = 0.142857$. Can you predict what the decimal expansions of $2/7$, $3/7$, $4/7$, $5/7$, $6/7$ are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $1/7$ carefully.]

Solution:

$$1/7 = 0.142857$$

$$\therefore 2 \times 1/7 = 2 \times 0.142857 = 0.285714$$

$$3 \times 1/7 = 3 \times 0.142857 = 0.428571$$

$$4 \times 1/7 = 4 \times 0.142857 = 0.571428$$

$$5 \times 1/7 = 5 \times 0.142857 = 0.714285$$

$$6 \times 1/7 = 6 \times 0.142857 = 0.857142$$

3. Express the following in the form p/q , where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$

Solution:

$$0.\overline{6} = 0.666...$$

Assume that $x = 0.666...$

$$\text{Then, } 10x = 6.666...$$

$$10x = 6 + x$$

$$9x = 6$$

$$x = 2/3$$

(ii)

Solution:

$$= (4/10) + (0.777/10)$$

Assume that $x = 0.777...$

Then, $10x = 7.777...$

$$10x = 7 + x$$

$$x = 7/9$$

$$(4/10) + (0.777.../10) = (4/10) + (7/90) \quad (x = 7/9 \text{ and } x = 0.777...0.777.../10 = 7/(9 \times 10) = 7/90)$$

$$= (36/90) + (7/90) = 43/90$$

$$(iii) 0.\overline{001}$$

Solution:

$$0.\overline{001} = 0.001001...$$

Assume that $x = 0.001001...$

Then, $1000x = 1.001001...$

$$1000x = 1 + x$$

$$999x = 1$$

$$x = 1/999$$

4. Express 0.99999.... in the form p/q . Are you surprised by your answer?

With your teacher and classmates discuss why the answer makes sense.

Solution:

Assume that $x = 0.9999....$ Eq. (a)

Multiplying both sides by 10,

$$10x = 9.9999.... \text{ Eq. (b)}$$

Eq.(b) - Eq.(a), we get

$$10x = 9.9999$$

$$-x = -0.9999...$$

$$9x = 9$$

$$x = 1$$

The difference between 1 and 0.999999 is 0.000001 which is negligible.

Hence, we can conclude that, 0.999 is too much near 1, therefore, 1 as the answer can be justified.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $1/17$? Perform the division to check your answer.

Solution:

$$1/17$$

Dividing 1 by 17:

0.0588235294117647

$$\begin{array}{r}
 17 \overline{) 100} \\
 \underline{85} \\
 150 \\
 \underline{136} \\
 140 \\
 \underline{136} \\
 40 \\
 \underline{34} \\
 60 \\
 \underline{51} \\
 90 \\
 \underline{85} \\
 50 \\
 \underline{34} \\
 160 \\
 \underline{153} \\
 70 \\
 \underline{68} \\
 20 \\
 \underline{17} \\
 30 \\
 \underline{17} \\
 130 \\
 \underline{119} \\
 110 \\
 \underline{102} \\
 80 \\
 \underline{68} \\
 120 \\
 \underline{119} \\
 100
 \end{array}$$

$$\frac{1}{17} = 0.0588235294117647$$

There are 16 digits in the repeating block of the decimal expansion of $1/17$.

6. Look at several examples of rational numbers in the form p/q ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solution:

We observe that when q is 2, 4, 5, 8, 10... Then the decimal expansion is terminating. For example:

$$1/2 = 0.5, \text{ denominator } q = 2^1$$

$$7/8 = 0.875, \text{ denominator } q = 2^3$$

$$4/5 = 0.8, \text{ denominator } q = 5^1$$

We can observe that the terminating decimal may be obtained in the situation where prime factorization of the denominator of the given fractions has the power of only 2 or only 5 or both.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution:

We know that all irrational numbers are non-terminating non-recurring. three numbers with decimal expansions that are non-terminating non-recurring are:

1. $\sqrt{3} = 1.732050807568$

2. $\sqrt{26} = 5.099019513592$

3. $\sqrt{101} = 10.04987562112$

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Solution:

$$\frac{5}{7} = 0.\overline{714285}$$

$$\frac{9}{11} = 0.\overline{81}$$

Three different irrational numbers are:

1. $0.73073007300073000073\dots$

2. $0.75075007300075000075\dots$

3. $0.76076007600076000076\dots$

9. Classify the following numbers as rational or irrational according to their type:

(i) $\sqrt{23}$

Solution:

$$\sqrt{23} = 4.79583152331\dots$$

Since the number is non-terminating and non-recurring therefore, it is an irrational number.

(ii) $\sqrt{225}$

Solution:

$$\sqrt{225} = 15 = \frac{15}{1}$$

Since the number can be represented in p/q form, it is a rational number.

(iii) 0.3796

Solution:

Since the number, 0.3796 , is terminating, it is a rational number.

(iv) 7.478478

Solution:

The number, 7.478478, is non-terminating but recurring, it is a rational number.

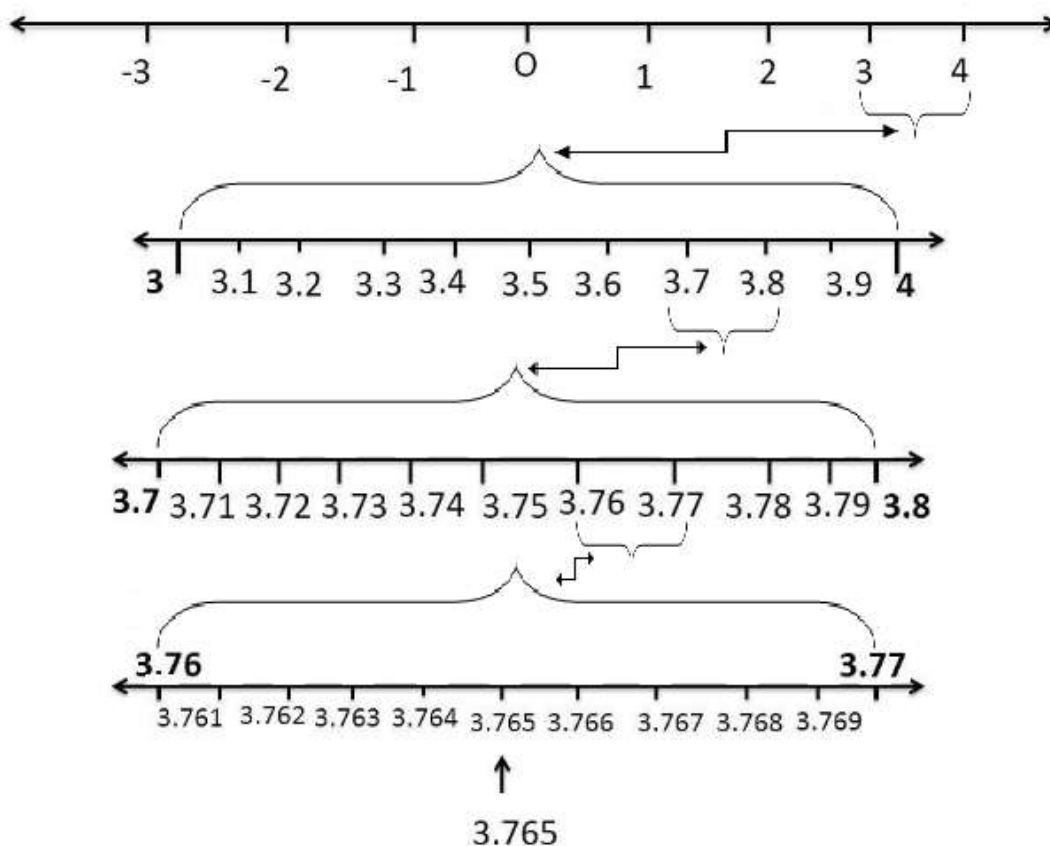
(v) 1.101001000100001...

Solution:

Since the number, 1.101001000100001..., is non-terminating non-repeating (non-recurring), it is an irrational number.

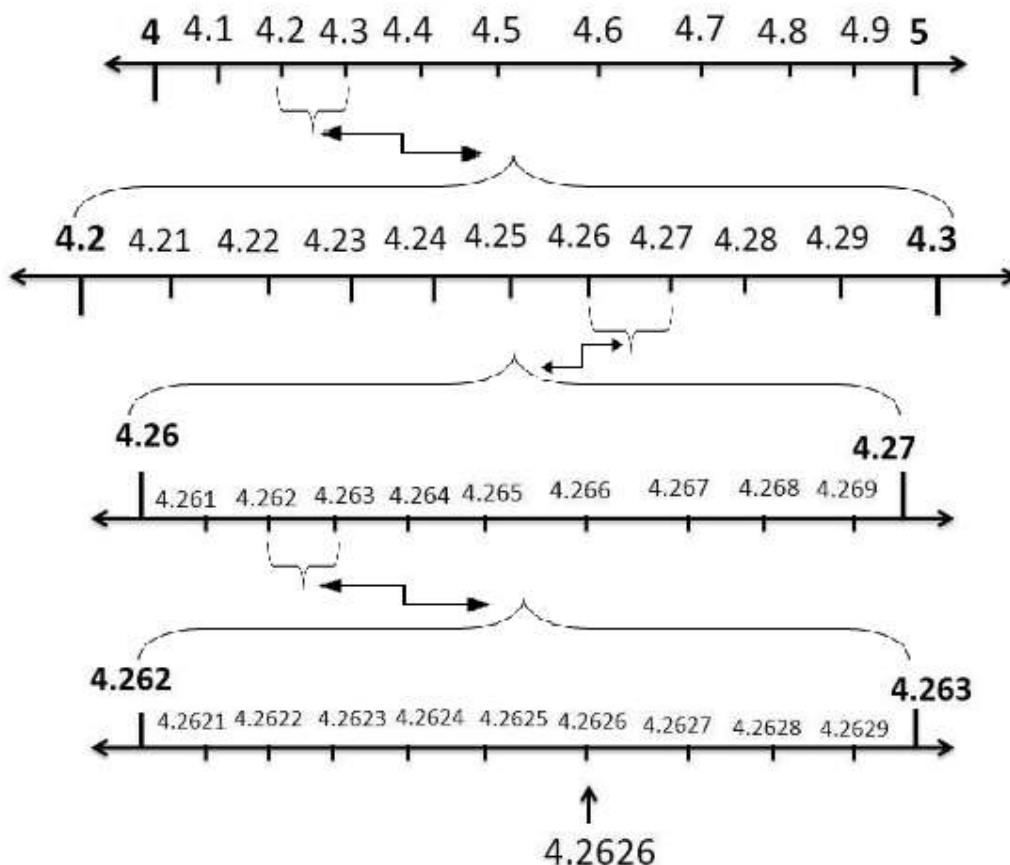
Exercise 1.4 Page: 18**1. Visualise 3.765 on the number line, using successive magnification.**

Solution:

**2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.****Solution:**

$$4.\overline{26} = 4.26262626\dots$$

$$4.\overline{26} \text{ up to 4 decimal places} = 4.2626$$



Exercise 1.5 Page: 24

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$

Solution:

We know that, $\sqrt{5} = 2.2360679...$

Here, $2.2360679...$ is non-terminating and non-recurring.

Now, substituting the value of $\sqrt{5}$ in $2 - \sqrt{5}$, we get,

$$2 - \sqrt{5} = 2 - 2.2360679... = -0.2360679$$

Since the number, $-0.2360679...$, is non-terminating non-recurring, $2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

Solution:

$$(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23}$$

$$= 3$$

$$= 3/1$$

Since the number $3/1$ is in p/q form, $(3 + \sqrt{23}) - \sqrt{23}$ is rational.

(iii) $2\sqrt{7}/7\sqrt{7}$

Solution:

$$2\sqrt{7}/7\sqrt{7} = (2/7) \times (\sqrt{7}/\sqrt{7})$$

We know that $(\sqrt{7}/\sqrt{7}) = 1$

$$\text{Hence, } (2/7) \times (\sqrt{7}/\sqrt{7}) = (2/7) \times 1 = 2/7$$

Since the number, $2/7$ is in p/q form, $2\sqrt{7}/7\sqrt{7}$ is rational.

(iv) $1/\sqrt{2}$

Solution:

Multiplying and dividing numerator and denominator by $\sqrt{2}$ we get,

$$(1/\sqrt{2}) \times (\sqrt{2}/\sqrt{2}) = \sqrt{2}/2 \text{ (since } \sqrt{2} \times \sqrt{2} = 2)$$

We know that, $\sqrt{2} = 1.4142...$

$$\text{Then, } \sqrt{2}/2 = 1.4142/2 = 0.7071..$$

Since the number, $0.7071..$ is non-terminating non-recurring, $1/\sqrt{2}$ is an irrational number.

(v) 2

Solution:

We know that, the value of $\sqrt{2} = 3.1415$

$$\text{Hence, } 2 = 2 \times 3.1415.. = 6.2830...$$

Since the number, $6.2830...$, is non-terminating non-recurring, 2 is an irrational number.

2. Simplify each of the following expressions:

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$

Solution:

$$(3 + \sqrt{3})(2 + \sqrt{2})$$

Opening the brackets, we get, $(3 \times 2) + (3 \times \sqrt{2}) + (\sqrt{3} \times 2) + (\sqrt{3} \times \sqrt{2})$

$$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$

Solution:

$$(3 + \sqrt{3})(3 - \sqrt{3}) = 3^2 - (\sqrt{3})^2 = 9 - 3$$

$$= 6$$

(iii) $(\sqrt{5} + \sqrt{2})^2$

Solution:

$$(\sqrt{5} + \sqrt{2})^2 = \sqrt{5}^2 + (2 \times \sqrt{5} \times \sqrt{2}) + \sqrt{2}^2$$

$$= 5 + 2 \times \sqrt{10} + 2 = 7 + 2\sqrt{10}$$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Solution:

$$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5^2}-\sqrt{2^2}) = 5-2 = 3$$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter, (say d). That is, $\pi = c/d$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Solution:

There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realize whether c or d is irrational. The value of π is almost equal to $22/7$ or $3.142857...$

4. Represent $(\sqrt{9.3})$ on the number line.

Solution:

Step 1: Draw a 9.3 units long line segment, AB. Extend AB to C such that $BC=1$ unit.

Step 2: Now, $AC = 10.3$ units. Let the centre of AC be O.

Step 3: Draw a semi-circle of radius OC with centre O.

Step 4: Draw a BD perpendicular to AC at point B intersecting the semicircle at D. Join OD.

Step 5: OBD, obtained, is a right angled triangle.

Here, $OD = 10.3/2$ (radius of semi-circle), $OC = 10.3/2$, $BC = 1$

$OB = OC - BC$

$$\Rightarrow (10.3/2) - 1 = 8.3/2$$

Using Pythagoras theorem,

We get,

$$OD^2 = BD^2 + OB^2$$

$$\Rightarrow (10.3/2)^2 = BD^2 + (8.3/2)^2$$

$$\Rightarrow BD^2 = (10.3/2)^2 - (8.3/2)^2$$

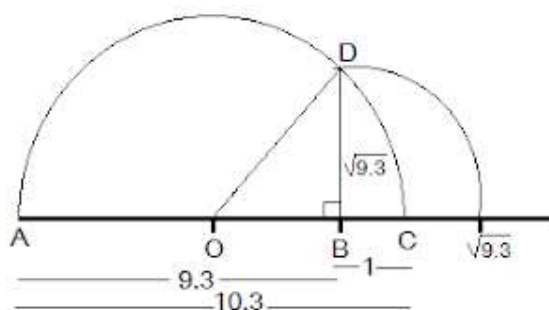
$$\Rightarrow (BD)^2 = (10.3/2) \cdot (10.3/2) - (8.3/2) \cdot (8.3/2)$$

$$\Rightarrow BD^2 = 9.3$$

$$\Rightarrow BD = \sqrt{9.3}$$

Thus, the length of BD is $\sqrt{9.3}$.

Step 6: Taking BD as radius and B as centre draw an arc which touches the line segment. The point where it touches the line segment is at a distance of $\sqrt{9.3}$ from O as shown in the figure.



5. Rationalize the denominators of the following:

(i) $1/\sqrt{7}$

Solution:

Multiply and divide $1/\sqrt{7}$ by $\sqrt{7}$

$$(1 \times \sqrt{7}) / (\sqrt{7} \times \sqrt{7}) = \sqrt{7}/7$$

(ii) $1/(\sqrt{7}-\sqrt{6})$

Solution:

Multiply and divide $1/(\sqrt{7}-\sqrt{6})$ by $(\sqrt{7}+\sqrt{6})$

$$\begin{aligned} [1/(\sqrt{7}-\sqrt{6})] \times (\sqrt{7}+\sqrt{6}) / (\sqrt{7}+\sqrt{6}) &= (\sqrt{7}+\sqrt{6}) / (\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6}) \\ &= (\sqrt{7}+\sqrt{6}) / \sqrt{7^2-\sqrt{6}^2} \text{ [denominator is obtained by the property, } (a+b)(a-b) \\ &= a^2-b^2] \end{aligned}$$

$$= (\sqrt{7}+\sqrt{6}) / (7-6)$$

$$= (\sqrt{7}+\sqrt{6}) / 1$$

$$= \sqrt{7}+\sqrt{6}$$

(iii) $1/(\sqrt{5}+\sqrt{2})$

Solution:

Multiply and divide $1/(\sqrt{5}+\sqrt{2})$ by $(\sqrt{5}-\sqrt{2})$

$$\begin{aligned} [1/(\sqrt{5}+\sqrt{2})] \times (\sqrt{5}-\sqrt{2}) / (\sqrt{5}-\sqrt{2}) &= (\sqrt{5}-\sqrt{2}) / (\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2}) \\ &= (\sqrt{5}-\sqrt{2}) / (\sqrt{5^2-\sqrt{2}^2}) \text{ [denominator is obtained by the property, } (a+b)(a-b) \\ &= a^2-b^2] \end{aligned}$$

$$= (\sqrt{5}-\sqrt{2}) / (5-2)$$

$$= (\sqrt{5}-\sqrt{2}) / 3$$

(iv) $1/(\sqrt{7}-2)$

Solution:

Multiply and divide $1/(\sqrt{7}-2)$ by $(\sqrt{7}+2)$

$$\begin{aligned} 1/(\sqrt{7}-2) \times (\sqrt{7}+2) / (\sqrt{7}+2) &= (\sqrt{7}+2) / (\sqrt{7}-2)(\sqrt{7}+2) \\ &= (\sqrt{7}+2) / (\sqrt{7^2-2^2}) \text{ [denominator is obtained by the property, } (a+b)(a-b) \\ &= a^2-b^2] \end{aligned}$$

$$= (\sqrt{7}+2) / (7-4) = (\sqrt{7}+2) / 3$$

Exercise 1.6 Page: 26**1. Find:****(i) $64^{1/2}$** **Solution:**

$$\begin{aligned} 64^{1/2} &= (8 \times 8)^{1/2} \\ &= (8^2)^{1/2} \\ &= 8^1 [\because 2 \times 1/2 = 2/2 = 1] \\ &= 8 \end{aligned}$$

(ii) $32^{1/5}$ **Solution:**

$$\begin{aligned} 32^{1/5} &= (2^5)^{1/5} \\ &= (2^5)^{1/5} \\ &= 2^1 [\because 5 \times 1/5 = 1] \\ &= 2 \end{aligned}$$

(iii) $125^{1/3}$ **Solution:**

$$\begin{aligned} (125)^{1/3} &= (5 \times 5 \times 5)^{1/3} \\ &= (5^3)^{1/3} \\ &= 5^1 (3 \times 1/3 = 3/3 = 1) \\ &= 5 \end{aligned}$$

2. Find:**(i) $9^{3/2}$** **Solution:**

$$\begin{aligned} 9^{3/2} &= (3 \times 3)^{3/2} \\ &= (3^2)^{3/2} \\ &= 3^3 [\because 2 \times 3/2 = 3] \\ &= 27 \end{aligned}$$

(ii) $32^{2/5}$ **Solution:**

$$\begin{aligned} 32^{2/5} &= (2 \times 2 \times 2 \times 2 \times 2)^{2/5} \\ &= (2^5)^{2/5} \\ &= 2^2 [\because 5 \times 2/5 = 2] \\ &= 4 \end{aligned}$$

(iii) $16^{3/4}$ **Solution:**

$$16^{3/4} = (2 \times 2 \times 2 \times 2)^{3/4}$$

$$= (2^4)^{3/4}$$

$$= 2^3 [\because 4 \times 3/4 = 3]$$

$$= 8$$

$$\text{(iv) } 125^{-1/3}$$

$$125^{-1/3} = (5 \times 5 \times 5)^{-1/3}$$

$$= (5^3)^{-1/3}$$

$$= 5^{-1} [\because 3 \times -1/3 = -1]$$

$$= 1/5$$

3. Simplify:

$$\text{(i) } 2^{2/3} \times 2^{1/5}$$

Solution:

$$2^{2/3} \times 2^{1/5} = 2^{(2/3)+(1/5)} [\because \text{Since, } a^m \times a^n = a^{m+n} \text{ ___ Laws of exponents}]$$

$$= 2^{13/15} [\because 2/3 + 1/5 = (2 \times 5 + 3 \times 1)/(3 \times 5) = 13/15]$$

$$\text{(ii) } (1/3^3)^7$$

Solution:

$$(1/3^3)^7 = (3^{-3})^7 [\because \text{Since, } (a^m)^n = a^{m \times n} \text{ ___ Laws of exponents}]$$

$$= 3^{-21}$$

$$\text{(iii) } 11^{1/2} / 11^{1/4}$$

Solution:

$$11^{1/2} / 11^{1/4} = 11^{(1/2)-(1/4)}$$

$$= 11^{1/4} [\because (1/2) - (1/4) = (1 \times 4 - 2 \times 1)/(2 \times 4) = 4 - 2 / 8 = 2/8 = 1/4]$$

$$\text{(iv) } 7^{1/2} \times 8^{1/2}$$

Solution:

$$7^{1/2} \times 8^{1/2} = (7 \times 8)^{1/2} [\because \text{Since, } (a^m \times b^m) = (a \times b)^m \text{ ___ Laws of exponents}]$$

$$= 56^{1/2}$$
